MATH 291

L-2/T-1/IPE **Date: 07/09/2025**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-1 B. Sc. Engineering Examinations 2023-2024

Sub: **MATH 291** (Differential Equation, Vector Calculus and Laplace Transform)

Full Marks: 210 Time: 3 Hours

The figures in the margin indicate full marks

Symbols used have their usual meaning.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE** questions.

1. (a) Form an ODE of all parabolas whose axes are parallel to the x-axis and have latus rectum a. Also, write down the order, degree, and linearity of that differential equation. **(11)**

(b) Write Bernoulli's differential equation and the solving process for it. Also, solve **(12)**

$$\left(1-x^2\right)\frac{dy}{dx}+xy=xy^2.$$

(c) Archaeologists used pieces of burned wood, or char coal, found at the site to date prehistoric painting and drawings on walls and ceilings of a cave in Lascaux, France. The half-life of radioactive C-14 is approximately 5600 years. Determine the approximate age of a piece of burned wood if it was found that 85.5% of the C-14 found in living trees of the same type had decayed. **(12)**

2. (a) Define homogeneous and non-homogeneous differential equations with examples. Solve the initial value problem $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+2y=0,\ y\left(\frac{\pi}{4}\right)=2,\ y'\left(\frac{\pi}{4}\right)=-2$. **(11)**

(b) What are auxiliary equation, and general solution of a differential equation? Solve $\frac{d^2y}{dx^2}-2n\frac{dy}{dx}+n^2y=xe^{nx}\sin x$. Replace n by any positive integer. **(12)**

(c) Find the charge on the capacitor in an LRC series circuit at t = 0.01s when L = 0.05h, R = 2Ω, C = 0.01f, $E(t)$ = 0V, $q(0)$ = 5C, and $i(0)$ = 0A. Determine the first time at which the charge on the capacitor is equal to zero. **(12)**

3. (a) Define regular and irregular singular points of a differential equation with examples. Using the method of Frobenious, find the series solution of the differential equation $2x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+\left(x^2-3\right)y=0$. **(25)**

(b) Prove that $xJ_n'=-nJ_n+xJ_{n-1}$. **(10)**

4. (a) Show that $\frac{d}{dx}\left[J_0(x)\right]=J_1(x)$. **(11)**

(b) Define the Legendre differential equation and Legendre polynomial $P_n(x)$. Also, prove that $nP_n=(2n-1)xP_{n-1}-(n-1)P_{n-2}$. **(12)**

(c) Write the orthogonal properties of the Legendre polynomial and prove them. **(12)**

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SECTION – B

There are **FOUR** questions in this section. Answer any **THREE** questions.

5. (a) Evaluate following integrals using Laplace transform: **(17)**

   (i) $\int_0^\infty te^{-3t}\sin t\,dt$ (ii) $\int_0^\infty \frac{e^{-a}-e^{-b}}{t}dt$.

   (b) Show that (i) $L\{\sin\sqrt{t}\}=\frac{\sqrt{\pi}}{2s^{3/2}}e^{-1/4s}$ and (ii) find the Laplace transform of $J_0(t)$ where $J_0(t)$ is the Bessel function of order zero. **(18)**

6. (a) Find the inverse Laplace transform of $\frac{s}{(s^2+a^2)^2}$ by the use of convolution theorem. **(10)**

   (b) Using Laplace transform solve the initial value problems:

   (i) $\frac{d^2Y}{dt^2}-3\frac{dY}{dt}+2Y=4e^{2t},\ Y(0)=-3,\ \frac{dY(0)}{dt}=5$ **(12)**

   (ii) $\frac{d^2Y}{dt^2}-t\frac{dY}{dt}+Y=1,\ Y(0)=1,\ \frac{dY(0)}{dt}=2.$ **(13)**

7. (a) Define curvature of a space curve. Find the torsion of the curve defined by **(13)**

   $$x=\frac{2t+1}{t-1},\quad y=\frac{t^2}{t-1},\quad Z=t+2.$$

   (b) Suppose an *xyz*- coordinate system is located in space such that the temperature T at the point (x, y, z) is given by the formula $T=100/(x^2+y^2+z^2)$. **(14)**

   (i) Measure the rate of change of T with respect to distance at the point P(1, 3, –2) in the direction of the vector $\vec{a}=\hat{i}-\hat{j}+\hat{k}$.

   (ii) In what direction from P does T increase most rapidly? What is the maximum rate of change of T at P?

   (c) Is there a differentiable vector function $\vec{V}$ such that curl $\vec{V}=x\hat{i}+y\hat{j}+z\hat{k}$. Justify your answer. **(4)**

   (d) Determine the constant a so that the vector $\vec{V}=(x+3y)\hat{i}+(y-2z)\hat{j}+(x+az)\hat{k}$ is solenoidal. Also, calculate the curl of the vector field using found value of a. **(4)**

8. (a) State Green's theorem in the plane and Divergence theorem. **(8)**

   (b) Find the outward flux for $\vec{F}=4x\hat{i}-2y^2\hat{j}+z^2\hat{k}$ taken over the region bounded by $x^2+y^2=16$, $z=0$ and $z=5$. **(15)**

   (c) Use Stokes' theorem to evaluate $\int_C \vec{F}.d\vec{r}$, if $\vec{F}=xz\hat{i}+xy\hat{j}+3xz\hat{k}$ and C is the boundary of the portion of the plane $2x+y+z=2$ in the first octant, traversed counterclockwise. **(12)**

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